B-Trees

Κ08 Δομές Δεδομένων και Τεχνικές Προγραμματισμού

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Searching on disks

- So far we have assumed that our data are stored in memory
- What about storing data on a **disk**?
  - **Example**: databases
- **Disk access** can be at least 100,000 to 1,000,000 slower
  - Goal: **minimize disk accesses**
- Also: data is read in **blocks**
  - Eg 512 or 1024 bytes
  - Reading 1 byte is the same as **reading a whole block**

(a, b)-Trees

We can easily generalize $(2, 4)$-trees:

- An (a, b)**-tree** is a b-way search tree with 2 extra properties
- **Size property**
  - Each node contains between $a - 1$ **and** $b - 1$ **values**
    (so each **internal** node contains between a **and** b **children**)
  - The **root is excluded** from this rule
- **Depth property**
  - All **leaves** have the **same depth** (lie on the same level)
- For the algorithms to work we need $2 \leq a \leq \frac{(b+1)}{2}$

B-Trees

- A **B-tree** of order m is an (a,b)-tree with $a = \lceil \frac{m}{2} \rceil$ and $b = m$
- We select m so that the **whole B-tree node** fits in a block
  - We read “multiple nodes” for the “price” of one
  - Fewer disk accesses than reading multiple nodes of a BST / AVL / …
- Such trees are **balanced**
  - $h = O(\log n)$
  - Proof: exercise

ADT Set using BTree, types

```
// Κόμβος του set, περιέχει μία μόνο τιμή. Κάθε btree_node έχει πολλά
struct set_node {
    Pointer value;          // Η τιμή του κόμβου.
    BTreeNode owner;        // Ο btree_node στον οποίο ανήκει αυτό το
};

// Το struct btree_node είναι ο κόμβος ενός Β-Δέντρου.
struct btree_node {
    int count;                              // Αριθμός στοιχείων
    BTreeNode parent;                       // Πατέρας
    BTreeNode children[MAX_CHILDREN + 1];   // Παιδιά
    SetNode set_nodes[MAX_VALUES + 1];      // Δεδομένα (μέσα σε set
};

// Υλοποιούμε τον ADT Set μέσω B-Tree, οπότε το struct set είναι ένα
struct set {
    BTreeNode root;             // Η ρίζα του δέντρου
    int size;                   // Μέγεθος, για αποδοτικό set_size
    CompareFunc compare;        // Διάταξη
    DestroyFunc destroy_value;  // Συνάρτηση που καταστρέφει ένα στοι
};
```

Insertion in a B-tree

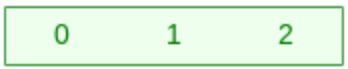
- Same as for 2-3 and 2-3-4-trees
  - Search for the value
  - Insert at a leaf
- In case of an overflow ($m + 1$ children)
  - Split it into two nodes of $m/2$ children each
  - Move the separator value (median) to the parent

Insert example, $m = 5$

Insert example, $m = 5$

Inserting 1

Insert example, $m = 5$

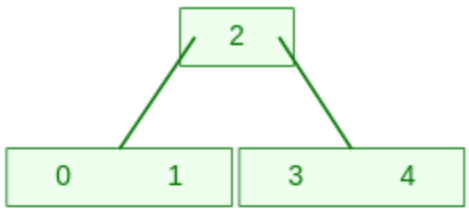


Inserting 2

Insert example, $m = 5$

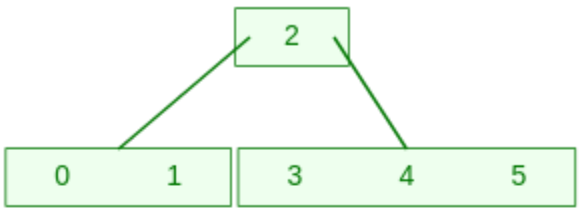
Inserting 3

Insert example, $m = 5$



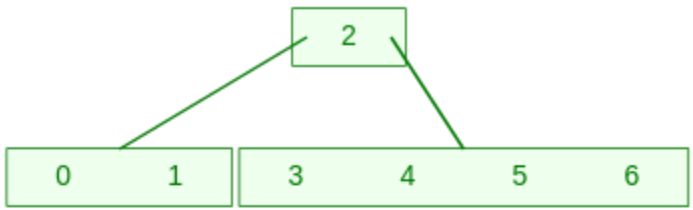
Inserting 4: overflow, 2 moves to a new root

Insert example, $m = 5$



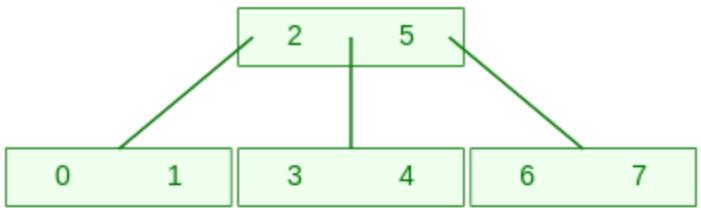
Inserting 5

Insert example, $m = 5$



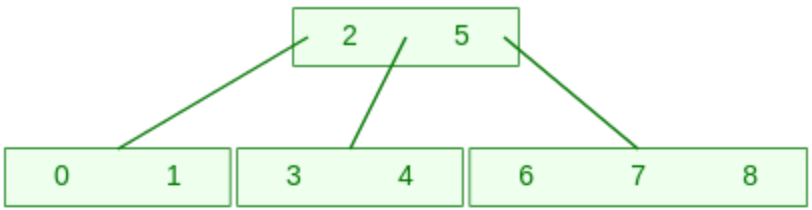
Inserting 6

Insert example, $m = 5$



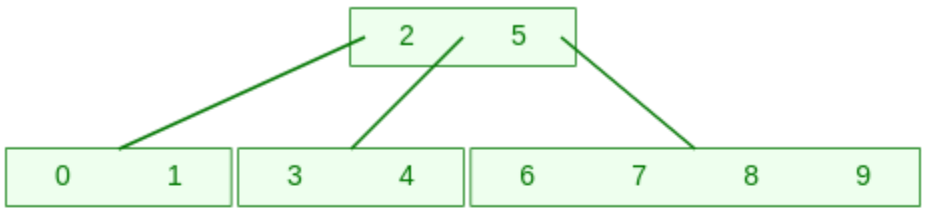
Inserting 7: overflow, 5 moves up

Insert example, $m = 5$



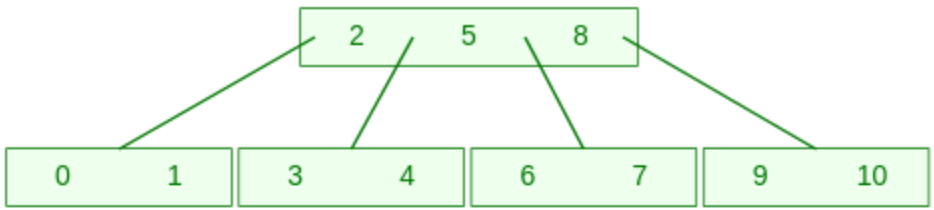
Inserting 8

Insert example, $m = 5$



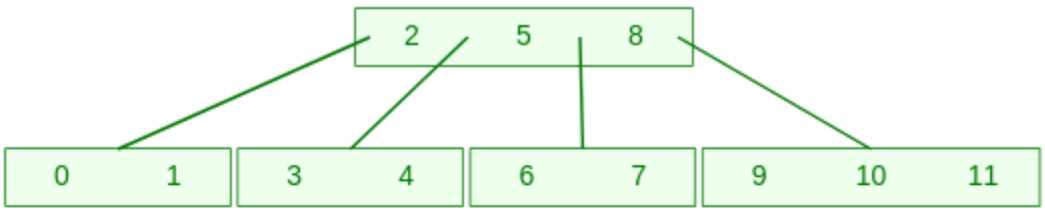
Inserting 9

Insert example, $m = 5$



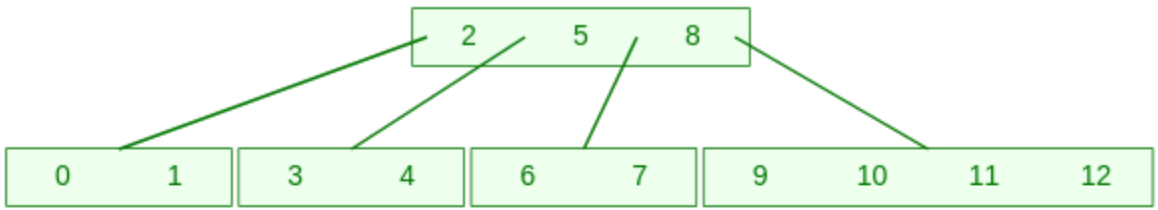
Inserting 10: overflow, 8 moves up

Insert example, $m = 5$



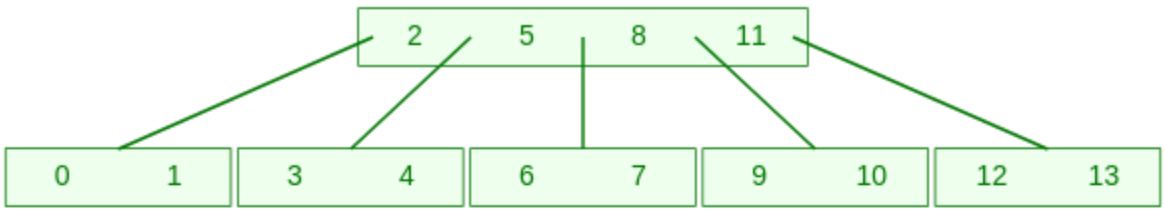
Inserting 11

Insert example, $m = 5$



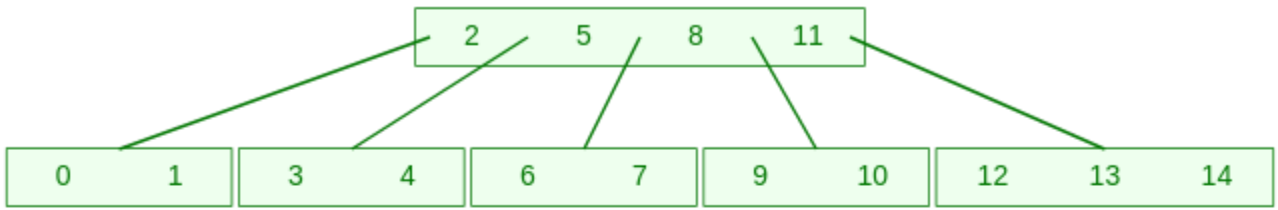
Inserting 12

Insert example, $m = 5$



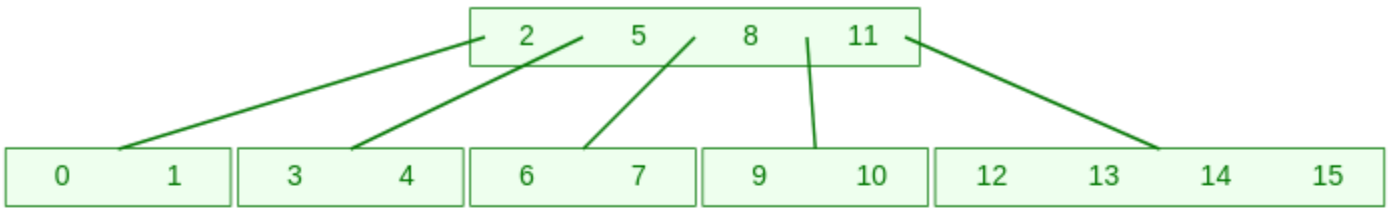
Inserting 13: overflow, 11 moves up

Insert example, $m = 5$



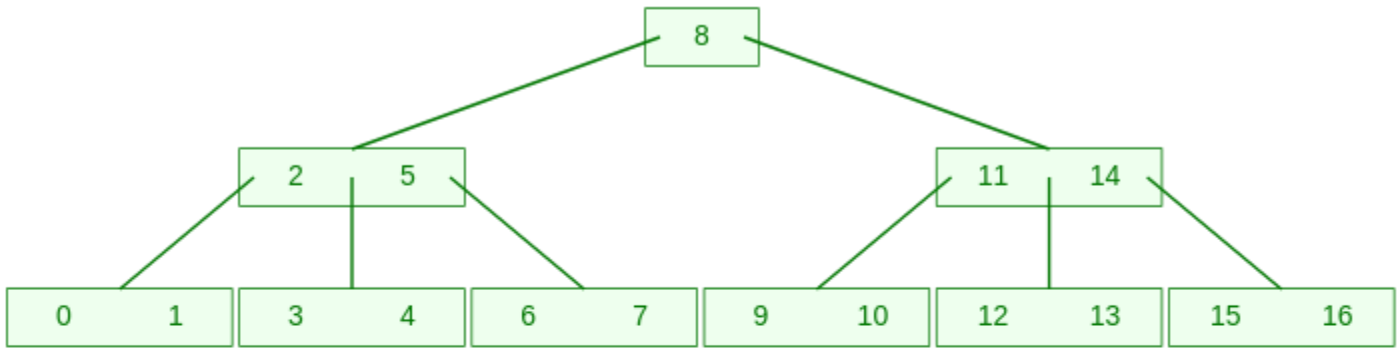
Inserting 14

Insert example, $m = 5$



Inserting 15

Insert example, $m = 5$



Inserting 16: overflow, 14 moves up, creating a new overflow

Code

- As always, the code is in `lecture-code`
  - `modules/UsingBTree/ADTSet.c`
- We only highlight some parts here

Code, `node_insert`

```
BTreeNode node_insert(BTreeNode root, CompareFunc compare, Pointer va
    // [απλός κώδικας για την περίπτωση κενού δέντρου]

    // Εύρεση του κόμβου στον οποίο πρέπει να γίνει insert
    int index;
    BTreeNode node = node_find(root, compare, value, &index);
    if (index != -1) {          // Υπάρχει ήδη η τιμή
        node->set_nodes[index]->value = value;
        return root;
    }

    // Εύρεση της θέσης που πρέπει να μπει το value
    for (index = 0;
         index < node->count && compare(value, node->set_nodes[index]
         index++)
    node_add_value(node, set_node_create(value), index);
        ;

    if (node->count > MAX_VALUES) // overflow
        split(node, compare);

    // Επιστρέφουμε τη ρίζα, μπορεί να έχει δημιουργηθεί νέα
    return root->parent != NULL ? root->parent : root;
}
```

Code, split

```
// Καλείται όταν ο κόμβος node έχει υπερχειλήσει, τον χωρίζει σε 2 κό
// Στέλνει τη μεσαία από τις τιμές του κόμβου node στον πατέρα του.

static void split(BTreeNode node, CompareFunc compare) {
    // Χωρίζουμε τον κόμβο node σε 2 κόμβους
    BTreeNode right = node_create();
    right->parent = node->parent;     // Οι 2 κόμβοι έχουν τον ίδιο π

    // Μετακίνησε τις μισές τιμές και παιδιά από τον αριστερό κόμβο σ
    int half = node->count/2;
    if (!is_leaf(node))
        for (int i = 0; i <= half; i++)
            node_add_child(right, node->children[i + half + 1], i);

    for (int i = 0; i < half; i++) {
        node_add_value(right, node->set_nodes[i + half + 1], i);
        node->count--;
    }

    // Αφαίρεση μεσαίας τιμής
    SetNode median = node->set_nodes[node->count-1];
    node->count--;

    ...
```

Code, split

```
    // Προσθέρουμε το median στον πατέρα του κόμβου node.
    BTreeNode parent = node->parent;
    if (parent == NULL) {                          // Ο node είναι η ρίζ
        BTreeNode new_root = node_create();        // Δημιούργησε καινού

        node_add_value(new_root, median, 0);

        right->parent = node->parent = new_root;
        new_root->children[0] = node;
        new_root->children[1] = right;

    } else {
        int index;  // Βρες τη θέση εισαγωγής της τιμής στον πατέρα.
        for (index = 0; index < parent->count; index++)
            if (compare(median->value, parent->set_nodes[index]->valu
                break;

        // Πρόσθεσε τον right ως δεξιό παιδί της (νέας) διαχωριστικής
        node_add_child(parent, right, index+1);
        node_add_value(parent, median, index);

        if (parent->count > MAX_VALUES)  // parent overflows
            split(parent, compare);
    }
}
```

Removal from B-trees

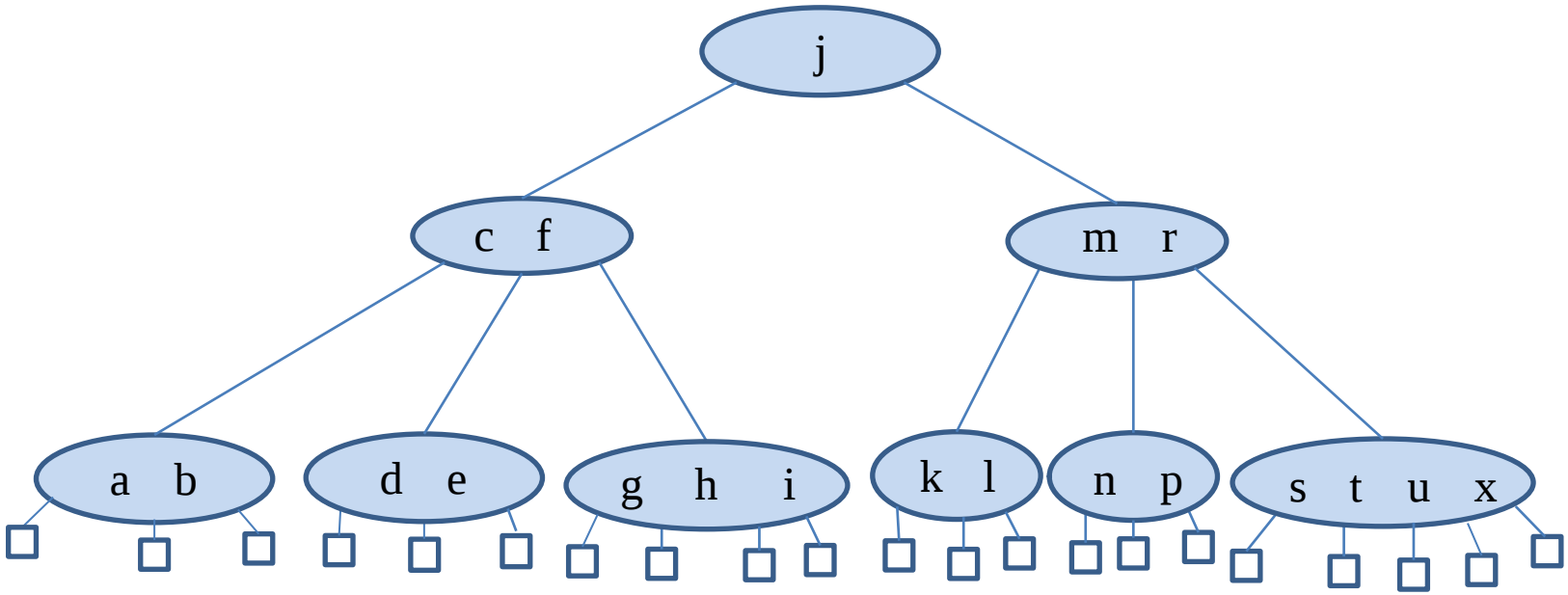
- Same as for 2-3 and 2-3-4-trees
- To remove a value k_i from an **internal** node
  - Replace with its **predecessor** (or its **successor**)
  - Right-most value in the i-th subtree
- To remove a value from a **leaf**
  - We simply remove it
  - But it might viotalate the **size** property (**underflow**)

Fixing underflows

Two strategies for fixing an underlow at ν

- Is there an **immediate sibling** w with a “spare” value?
- If so, we do a **transfer** operation
  - Move a value of w to its parent u
  - Move a value of the parent u to ν
- If not, we do a **fusion** operation
  - Merge ν and w, creating a new node ν'
  - Move a value from the parent u to ν'
  - This might **underflow the parent**, continue the same procedure there

Example



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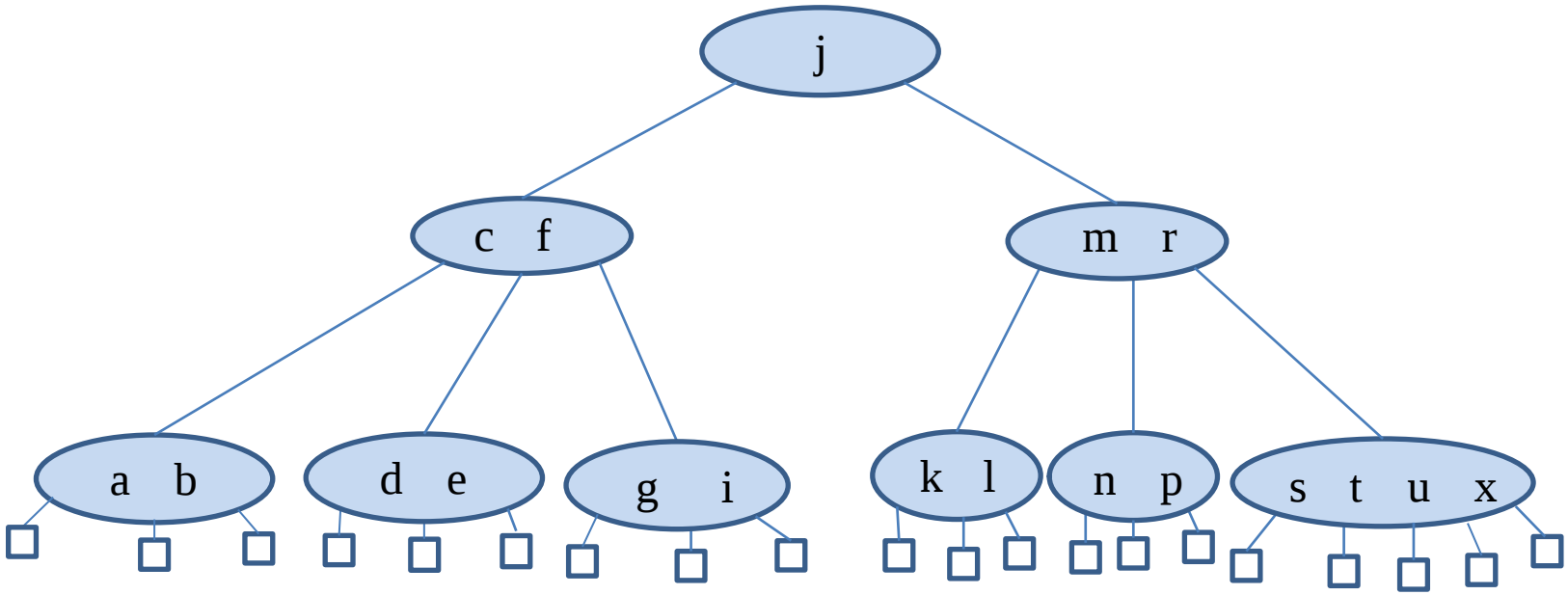
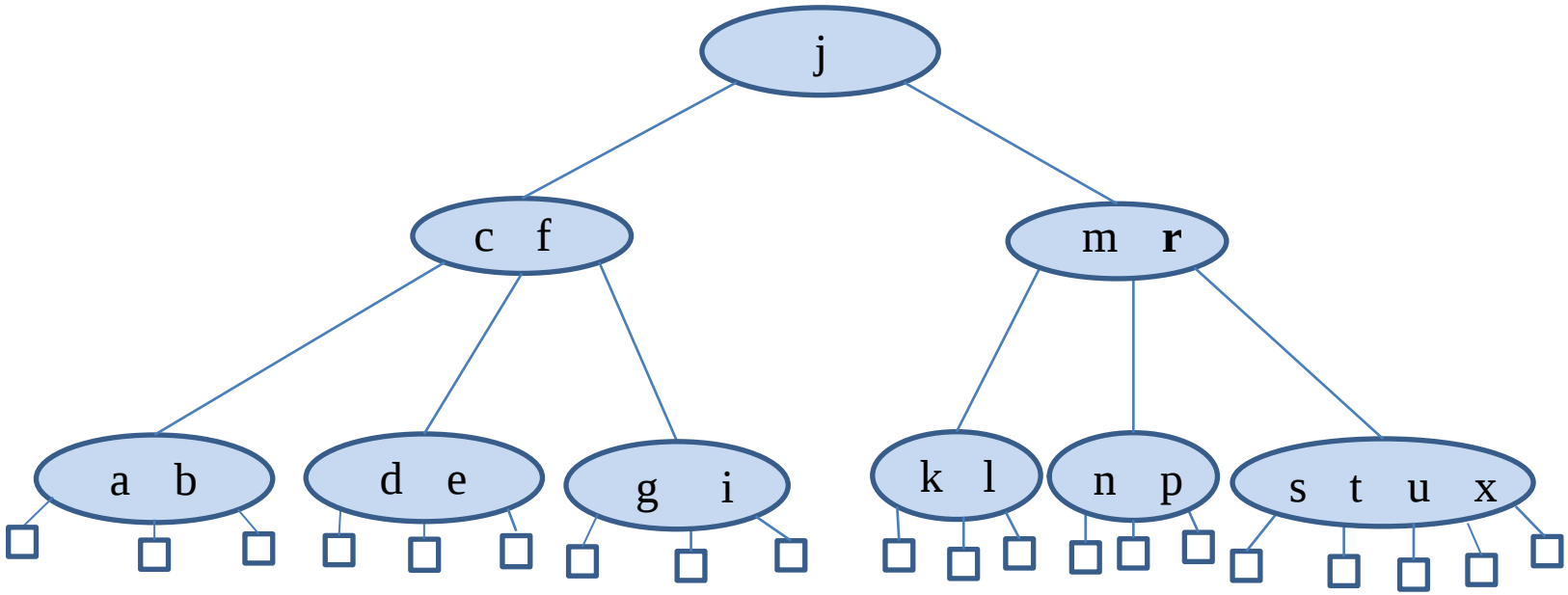
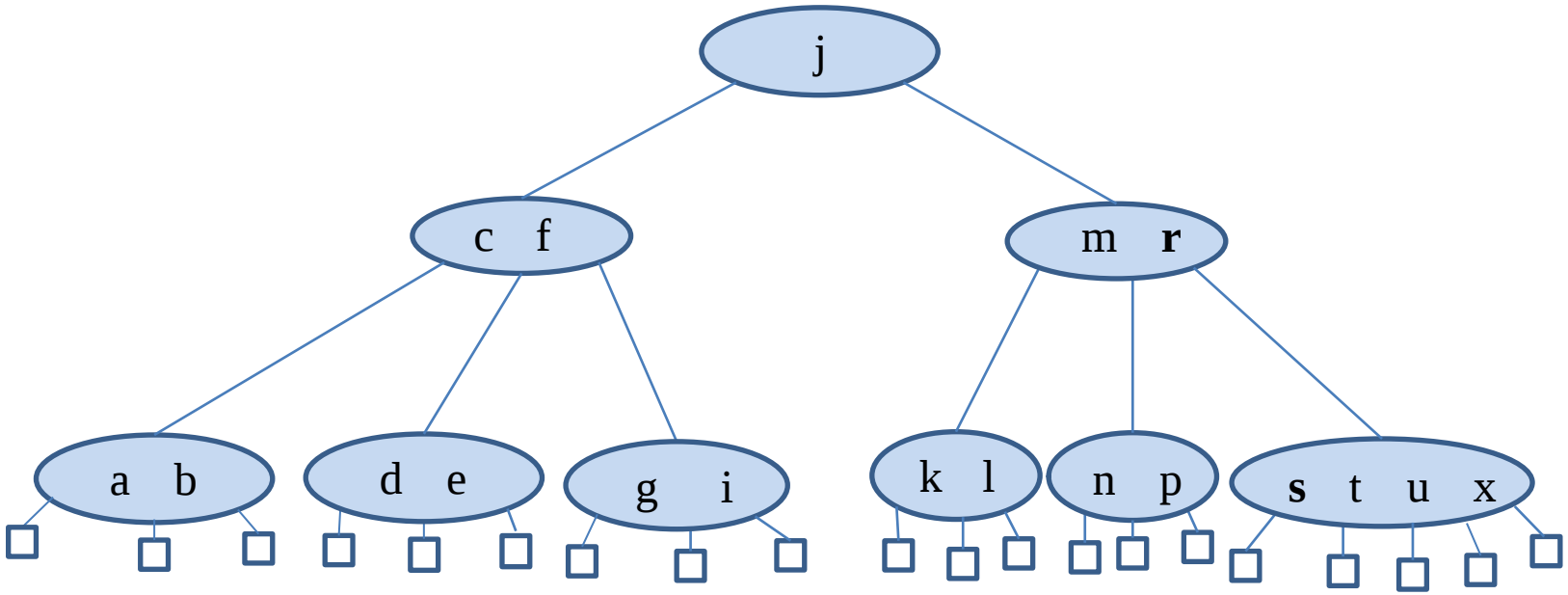


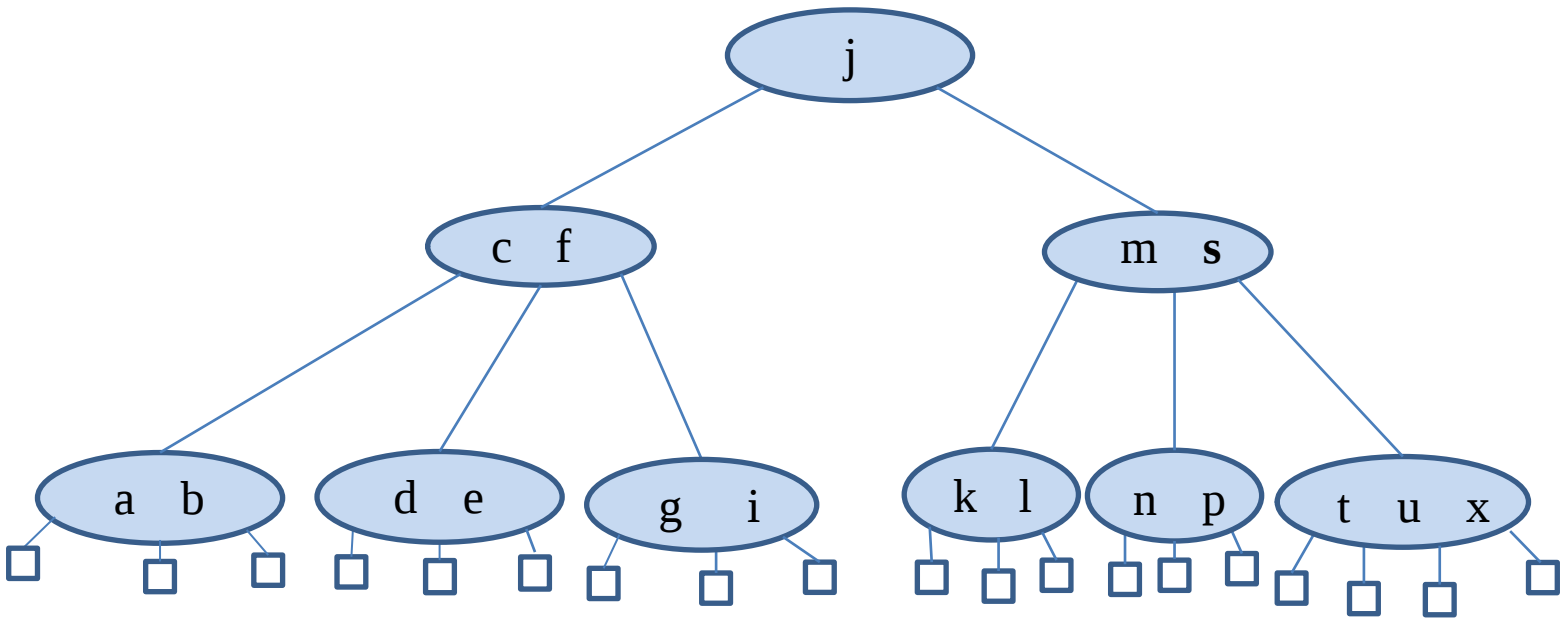
Delete r



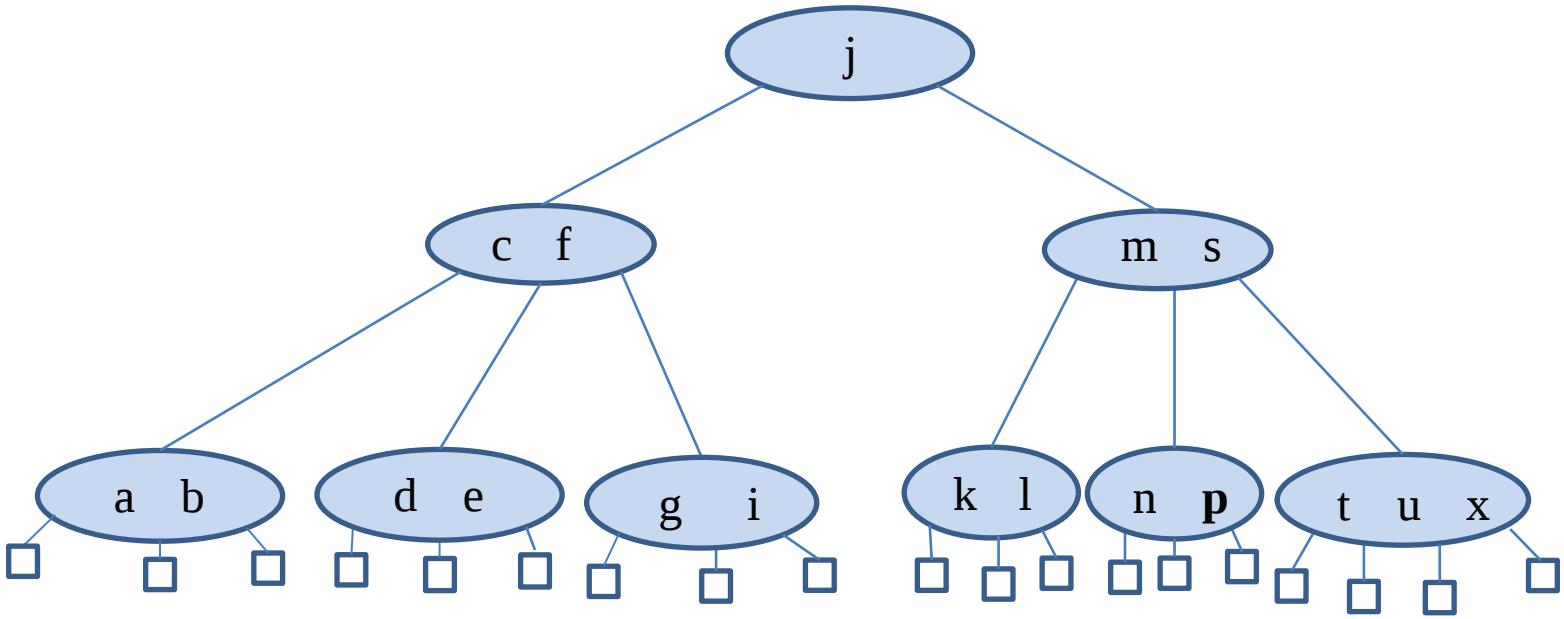
Find the Successor of r



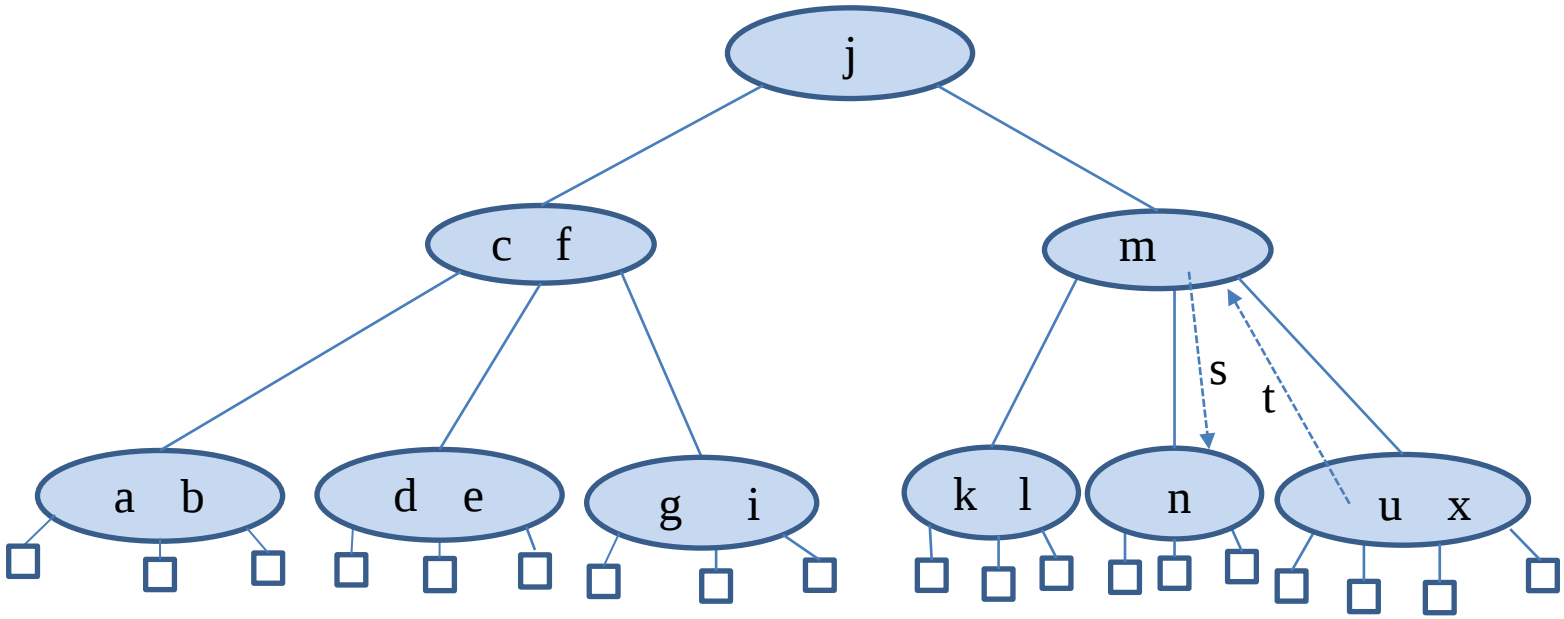
Promote the Successor of r – Delete the Successor from its Place



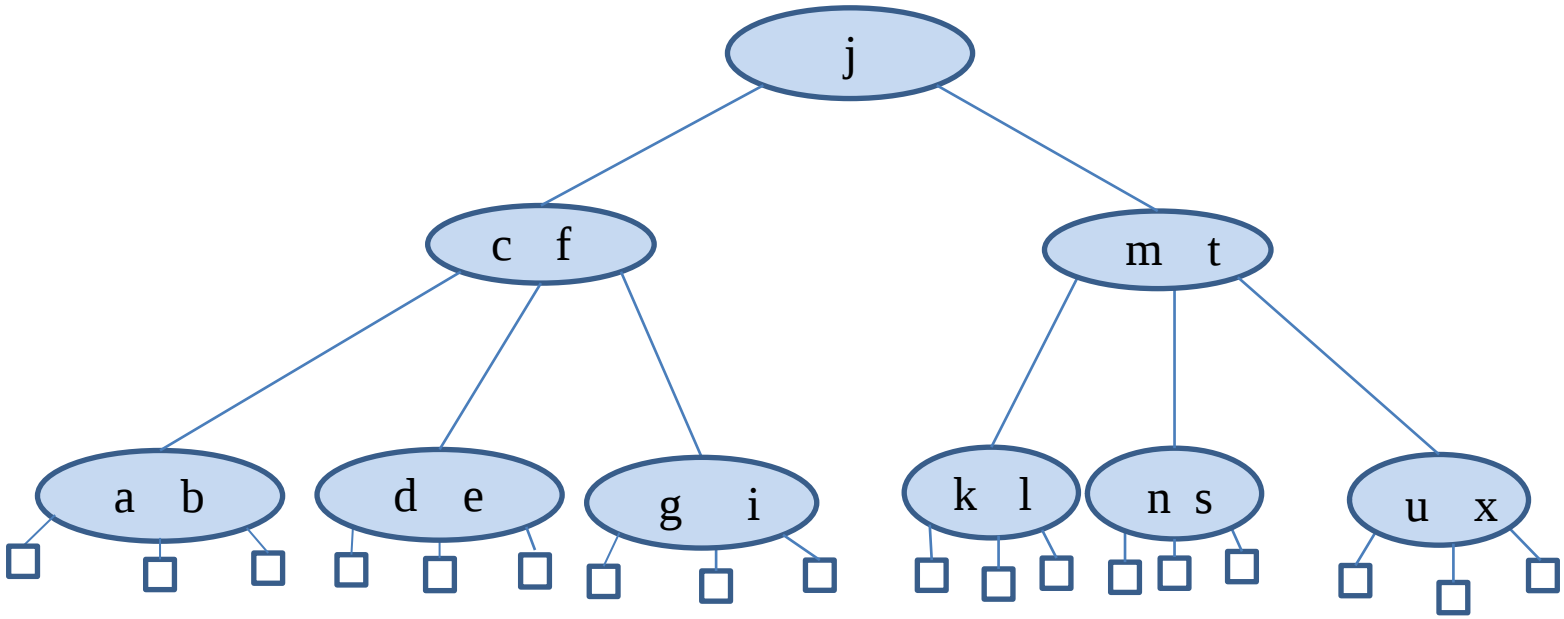
Delete p



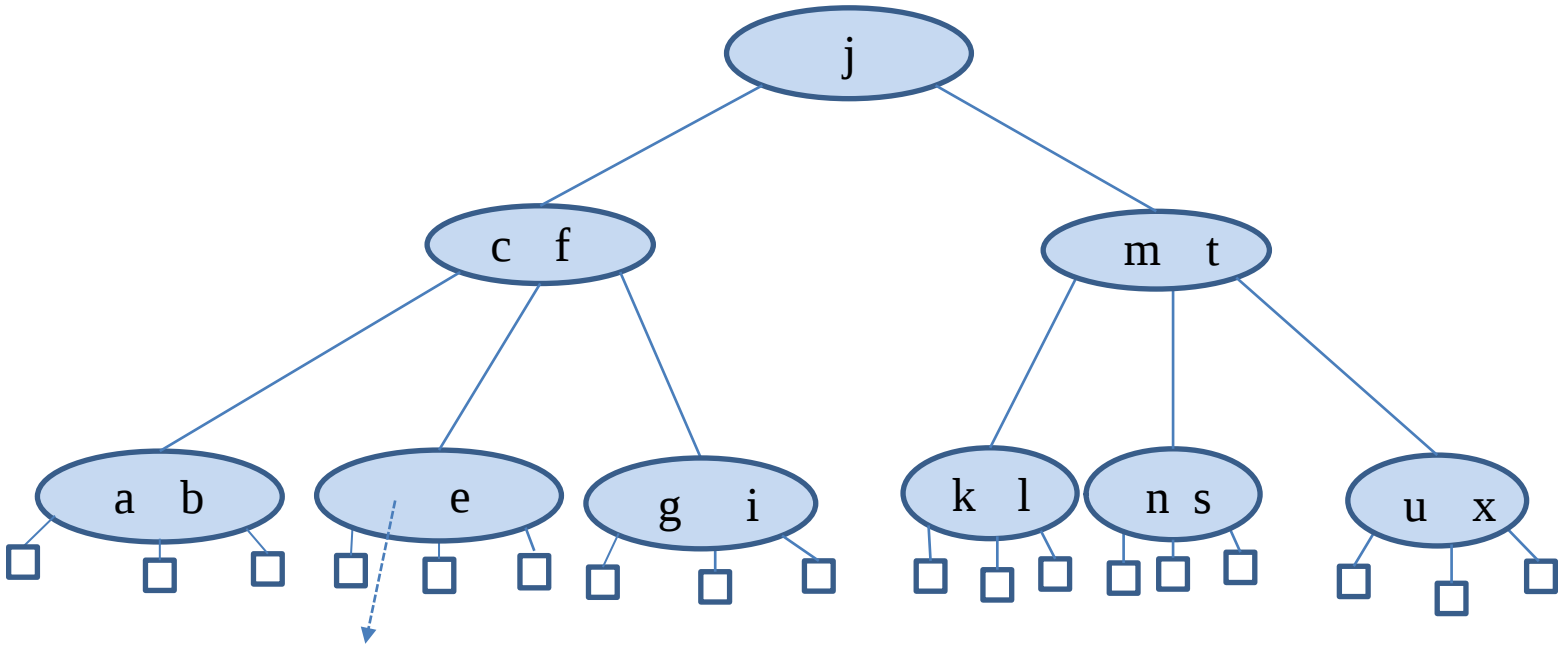
Transfer



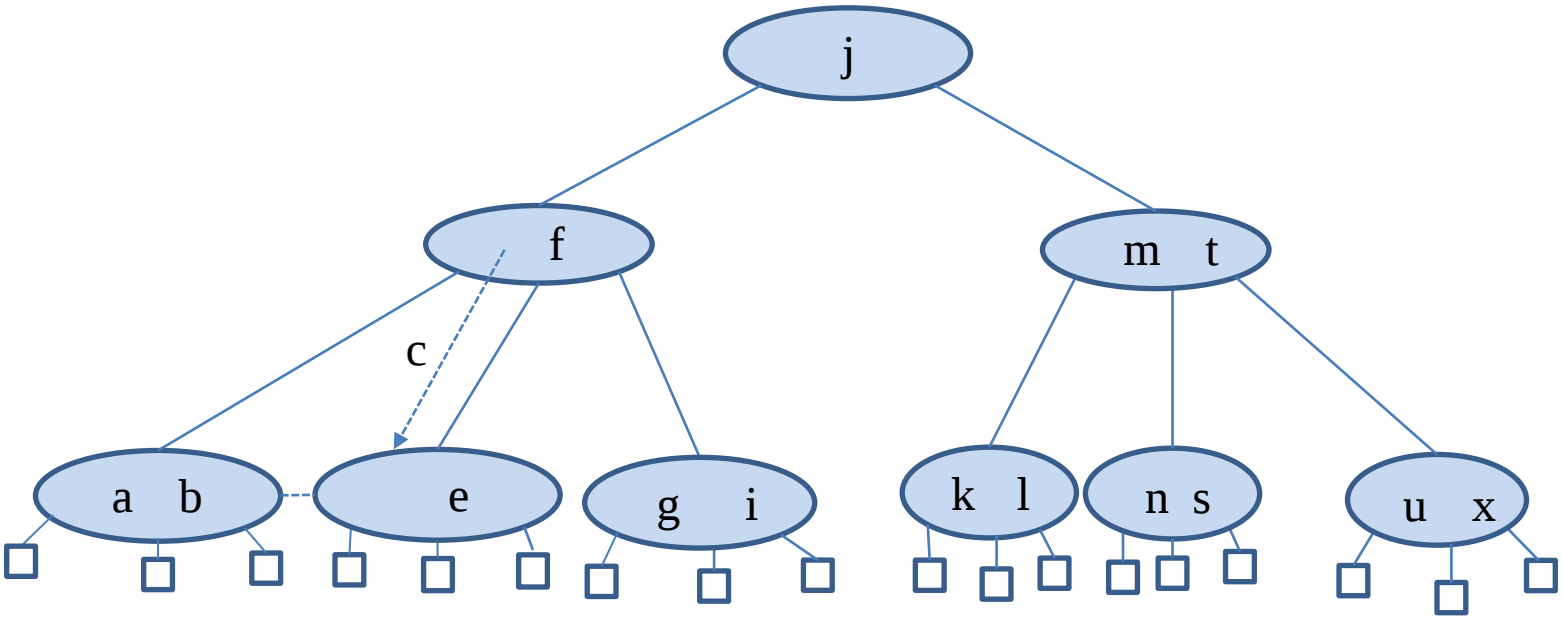
After the Transfer



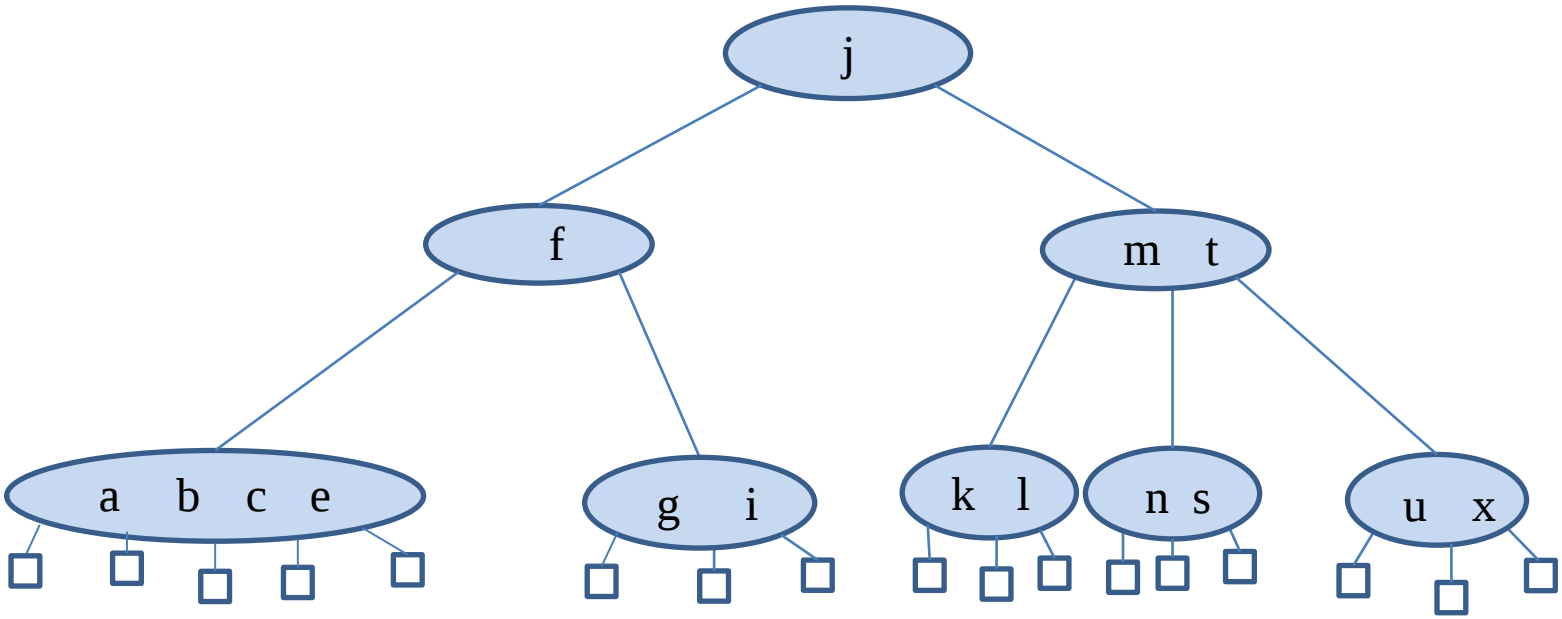
Delete d



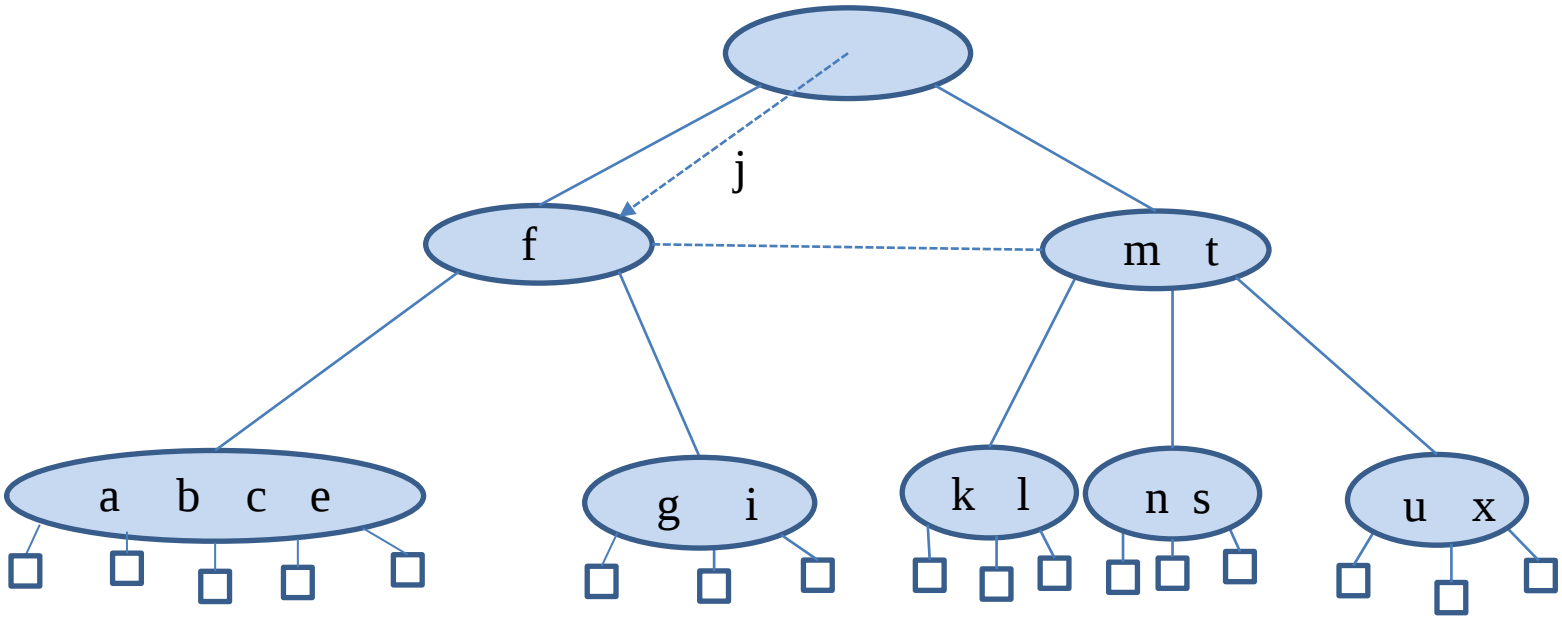
Fusion



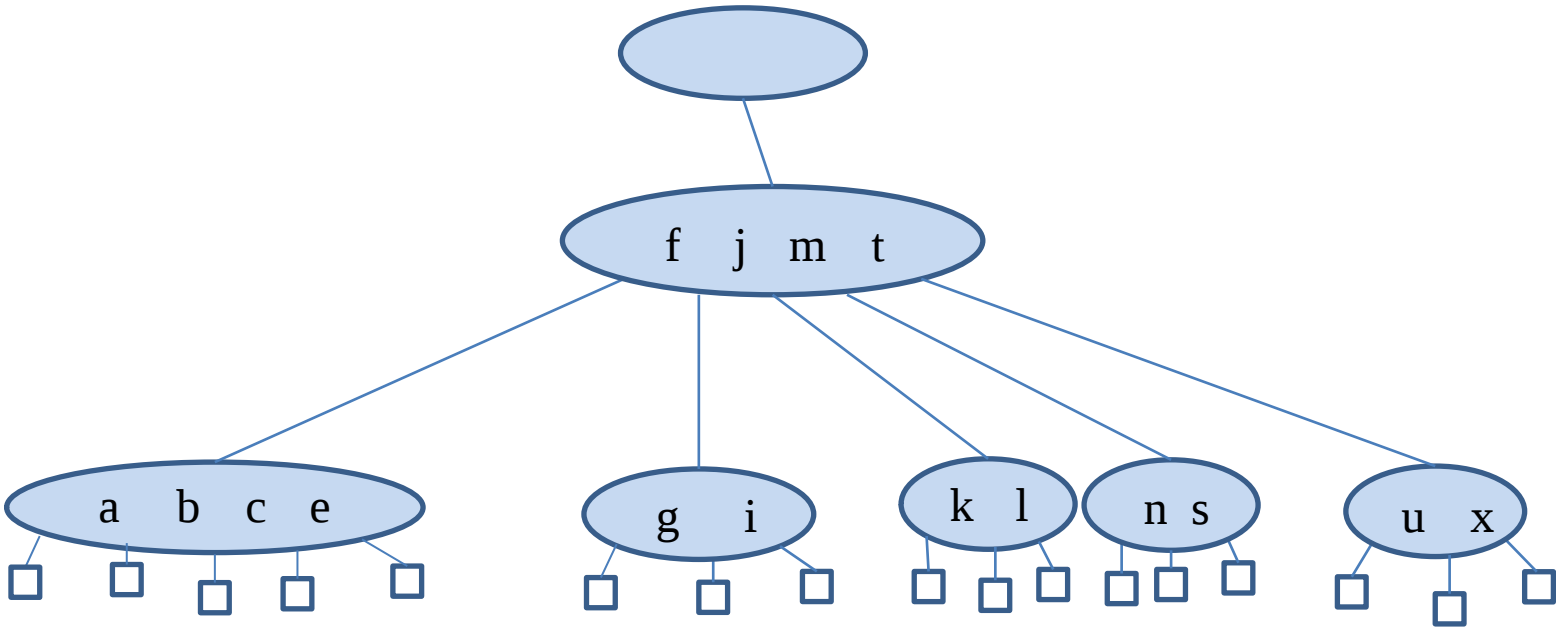
After the Fusion – Underflow at f



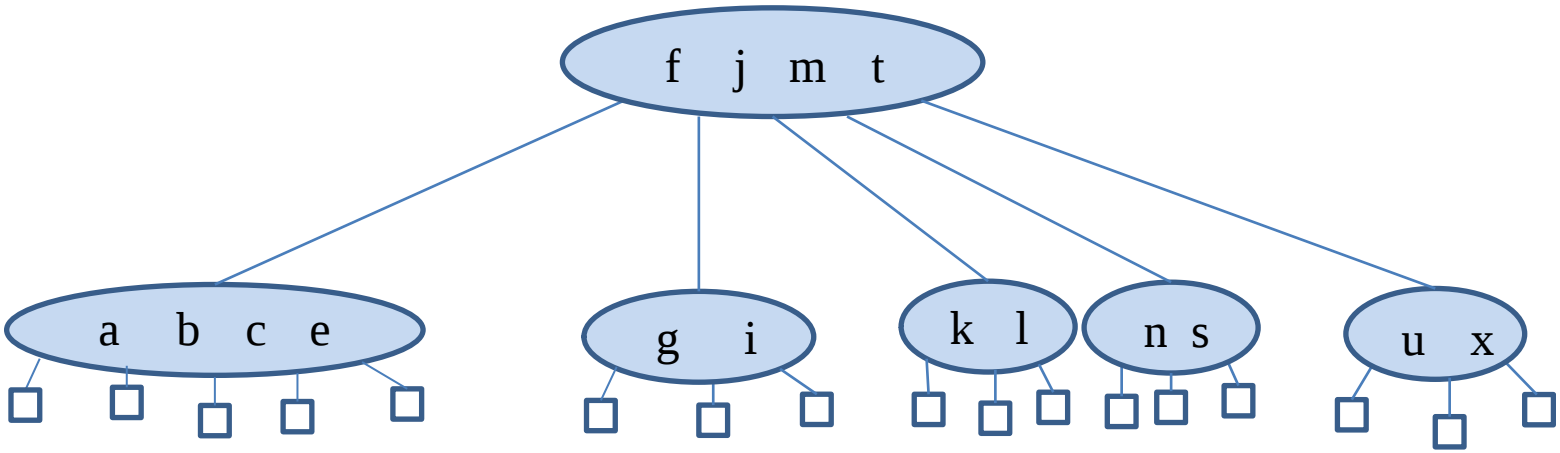
Fusion



After the Fusion – Delete Root



Final Tree



Code, node_remove

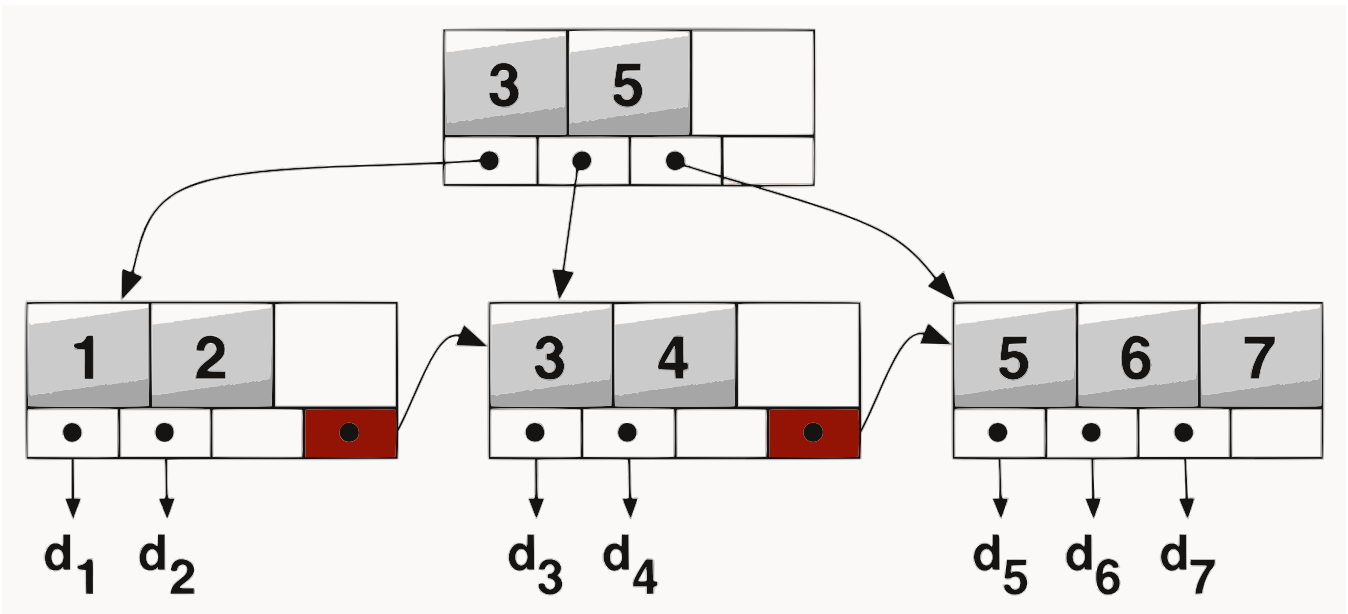
```
BTreeNode node_remove(BTreeNode root, CompareFunc compare, Pointer va
    // Βρες τον κόμβο που περιέχει την τιμή.
    int index;
    BTreeNode node = node_find(root, compare, value, &index);

    if (index == -1)   // Η τιμή δεν υπάρχει στο δέντρο.
        return root;

    // Βρέθηκε ισοδύναμη τιμή στον node, οπότε τον διαγράφουμε
    // Το πώς θα γίνει αυτό εξαρτάται από το αν έχει παιδιά.

    if (is_leaf(node)) {
        // Φύλλο: διάγραψε την τιμή, αναδιάταξε τα δεδομένα, repair
        // Ολίσθησε όλα τα δεδομένα 1 θέση αριστερά.
        for (int i = index; i < node->count-1; i++)
            node->set_nodes[i] = node->set_nodes[i + 1];

        node->count--;           // Αφαίρεσε το δεδομένο.

        repair_underflow(node);  // Αναδιαμόρφωσε το δένδρο.
```

Code, node_remove

```
    } else {
        // Άν είναι εσωτερικός κόμβος αντικατάσταση με την next τιμε
        // και remove της τιμής αυτής
        SetNode max = node_find_max(node->children[index]);

        BTreeNode max_node = max->owner;
        max_node->count--;    // Αφαίρεσε το δεδομένο.

        node->set_nodes[index] = max;
        max->owner = node;

        repair_underflow(max_node);   // Αναδιαμόρφωσε το δέντρο.
    }

    // Αν η ρίζα αδειάσει, ρίζα γίνεται το (μοναδικό, αν έχει) παιδί
    if (root->count == 0) {
        BTreeNode first_child = root->children[0];
        if (first_child != NULL)
            first_child->parent = NULL;
        root = first_child;
    }
    return root;
}
```

B+-trees

A variant of B-trees, important in today's **file systems** and **databases**.



Readings

- M. T. Goodrich, R. Tamassia and D. Mount. Data Structures and Algorithms in C++. 2nd edition. John Wiley.
- Sartaj Sahni. Δομές Δεδομένων, Αλγόριθμοι και Εφαρμογές στη C++. Εκδόσεις Τζιόλα.
- R. Sedgewick. Αλγόριθμοι σε C. Κεφ. 16.3