Dynamic Arrays

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How can we implement ADTVector?

• A Vector can be seen as an abstract resizable “array”

• So it makes sense to **implement** it using a **real array**
  - store Vector's elements in the array
  - `vector_get_at`, `vector_set_at` are trivial

• But what about `vector_insert_last`?
  - Arrays in C have fixed size
Dynamic arrays

• Main idea: **resize** the array
  - such arrays are called “dynamic” or “growable”

• **Problem**: we need to **copy** the previous values

• A possible algorithm for **vector_insert_last**
  - Allocate memory for **size+1** elements
  - Copy the **size** previous elements
  - Set the new element as last
  - Increase **size**

• What is the complexity of this?
  - $O(n)$, because of the copy!
  - Can we do better?
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Improving the complexity of insert

- **Idea**: allocate more memory than we need!
  - eg. allocate memory for 100 “empty” elements
    - **capacity**: total allocated memory
    - **size**: number of inserted elements
  - Insert is $O(1)$ if we have free space (just copy the new value)

- Does this change the complexity?
  - in the **worst-case**?
  - in the **average-case**?

- **No**, for some values of $n$ the operation is still slow!
  - For **any values**, “average-case” makes no difference
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Amortized-time complexity

- We see here the value of **amortized-time** complexity
  - A single execution **can** be slow
  - But “most” are fast
  - In many application we only care about the **average** wrt all **executions**

- Assume we reserve 100 more elements each time
  - How many steps each insert takes on average?

- Intuitively: $\frac{n}{100}$. So still $O(n)$, same complexity!
  - Same for any **constant** number of empty elements $k$
  - Remember, complexity cares about large $n$! Think $n \gg k$
  - Can we do better?
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How to improve the complexity

- **Idea**: the number of empty elements must depend on \( n \)
  - Use more empty elements as the Vector grows!
- Standard approach: reserve \( a \cdot n \) extra elements
  - for some constant \( a > 1 \), called the growth factor
- Common values
  - \( a = 2 \)
  - \( a = 1.5 \)
- In this class we will use \( a = 2 \)
  - we always double the capacity
A property to remember

• Consider the geometric progression with ratio 2

\[ 1, 2^1, 2^2, \ldots, 2^n \]

• Summing \( n \) terms, we get the next one minus 1

\[ 1 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1 \]

• So each term is larger than all the previous together!
  - This is important since several quantities double in data structures
From linear to constant time

• We always **double** the capacity
  - What is the amortized-time complexity of insert?

• We do \( n \) insertions starting from an empty Vector
  - Assume the last one was “slow” (the most “unlucky” case)

• How many **steps** did we perform **in total**?
  - \( n \) steps just for placing each element
  - \( n \) steps for the **last resize**
  - How many for all the previous resizes together?
    \[
    \frac{n}{2} + \frac{n}{4} + \ldots + 1 = n - 1
    \]

• So less than \( 3n \) in total!
  - On average: \( \frac{3n}{n} = O(1) \)
- Key point: previous inserts are insignificant compared to the last one
Removing elements

• What about \texttt{vector\_remove\_last}?

• Simplest strategy: just consider the removed space as “empty”
  - \texttt{vector\_remove\_last} is clearly worst-case $O(1)$
  - Insert is not affected (we never reduce the amount of free space)

• Commonly used in practice
  - eg. \texttt{std::vector} in C++

• \textbf{Problem}: wasted space
Recovering wasted space

- **Idea**: if half of the array becomes empty, resize
  - the opposite of the doubling growing strategy
  - Is this ok?

- Careful
  - this is ok if we only remove
  - but a combination of remove+insert might become slow!

- Think of the following scenario
  - Insert \( n \) elements with \( n = 2^k \)
  - The vector is now full
  - Perform a series of: insert, remove, insert, remove, …
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Recovering wasted space

• Better strategy
  - when only $\frac{1}{4}$ of the array is full
  - resize to $\frac{1}{2}$ of the capacity!
  - So we still have “room” to both insert and remove

• We can show that even a combination of insert+remove is $O(1)$ amortized-time
Implementation

Types

// Ένα VectorNode είναι pointer σε αυτό το struct.

```c
struct vector_node {
    Pointer value; // Η τιμή του κόμβου.
};
```

// Ένα Vector είναι pointer σε αυτό το struct

```c
struct vector {
    VectorNode array; // Τα δεδομένα, πίνακας από struct ve
    int size; // Πόσα στοιχεία έχουμε προσθέσει
    int capacity; // Πόσο χώρο έχουμε δεσμεύσει
    DestroyFunc destroy_value; // Συνάρτηση που καταστρέφει ένα στοι
};
```
Implementation

```c
Vector vector_create(int size, DestroyFunc destroy_value) {
    // Αρχικά το vector περιέχει size μη-αρχικοποιημένα στοιχεία, αλλά
    // δεσμεύουμε χώρο για τουλάχιστον VECTOR_MIN_CAPACITY για να απο
    // πολλαπλά resizes
    int capacity = size < VECTOR_MIN_CAPACITY ? VECTOR_MIN_CAPACITY :
    // Δέσμευση μνήμης, για το struct και το array.
    Vector vec = malloc(sizeof(*vec));
    VectorNode array = calloc(capacity, sizeof(*array)); // αρχικοπο
    vec->size = size;
    vec->capacity = capacity;
    vec->array = array;
    vec->destroy_value = destroy_value;

    return vec;
}
```
Implementation

Random access is simple, since we have a real array.

```c
Pointer vector_get_at(Vector vec, int pos) {
    return vec->array[pos].value;
}

void vector_set_at(Vector vec, int pos, Pointer value) {
    // Αν υπάρχει συνάρτηση destroy_value, την καλούμε για
    // το στοιχείο που αντικαθίσταται
    if (value != vec->array[pos].value && vec->destroy_value != NULL)
        vec->destroy_value(vec->array[pos].value);
    vec->array[pos].value = value;
}
```
Implementation

Insert, we just need to deal with resizes.

```c
void vector_insert_last(Vector vec, Pointer value) {
    // Μεγαλώνουμε τον πίνακα (αν χρειαστεί), ώστε να χωράει τουλάχιστον
    // στοιχεία. Διπλασιάζουμε κάθε φορά το capacity (σημαντικό για τη
    // πολυπλοκότητα!)
    if (vec->capacity == vec->size) {
        vec->capacity *= 2;
        vec->array = realloc(vec->array, vec->capacity * sizeof(*new_}
    }

    // Μεγαλώνουμε τον πίνακα και προσθέτουμε το στοιχείο
    vec->array[vec->size].value = value;
    vec->size++;
}
```
Takeaways

- **Dynamic arrays** are the standard way to implement ADTVector

- Insert is \( O(1) \)
  - but amortized-time!
  - would you use a dynamic array in the software controlling an Airbus?

- Remove is also \( O(1) \)
  - also amortized, if we care about recovering wasted space

- Random access (get/set) is always worst-case \( O(1) \)