Dynamic Arrays

K08 Δομές Δεδομένων και Τεχνικές Προγραμματισμού
Κώστας Χατζηκοκολάκης
How can we implement ADTVector?

• A Vector can be seen as an abstract resizable “array”

• So it makes sense to implement it using a real array
  - store Vector's elements in the array
  - `vector_get_at`, `vector_set_at` are trivial

• But what about `vector_insert_last`?
  - Arrays in C have fixed size
Dynamic arrays

• Main idea: **resize** the array
  - such arrays are called “dynamic” or “growable”

• **Problem**: we need to **copy** the previous values

• A possible algorithm for `vector_insert_last`
  - Allocate memory for `size+1` elements
  - Copy the `size` previous elements
  - Set the new element as last
  - Increase `size`

• What is the complexity of this?
  - $O(n)$, because of the copy!
  - Can we do better?
Dynamic arrays

- Main idea: **resize** the array
  - such arrays are called “dynamic” or “growable”

- **Problem**: we need to **copy** the previous values

- A possible algorithm for **vector_insert_last**
  - Allocate memory for **size+1** elements
  - Copy the **size** previous elements
  - Set the new element as last
  - Increase **size**

- What is the complexity of this?
  - $O(n)$, because of the copy!
  - Can we do better?
Improving the complexity of insert

- **Idea:** allocate **more memory** than we need!
  - eg. allocate memory for 100 “empty” elements
    - **capacity:** total allocated memory
    - **size:** number of inserted elements
  - Insert is $O(1)$ if we have free space (just copy the new value)

- Does this change the complexity?
  - in the **worst-case**?
  - in the **average-case**?

- **No,** for some values of $n$ the operation is still slow!
  - For **any values**, “average-case” makes no difference
Improving the complexity of insert

- Idea: allocate more memory than we need!
  - eg. allocate memory for 100 “empty” elements
    - capacity: total allocated memory
    - size: number of inserted elements
  - Insert is $O(1)$ if we have free space (just copy the new value)

- Does this change the complexity?
  - in the worst-case?
  - in the average-case?

- No, for some values of $n$ the operation is still slow!
  - For any values, “average-case” makes no difference
Amortized-time complexity

• We see here the value of **amortized-time** complexity
  - A single execution can be slow
  - But “most” are fast
  - In many application we only care about the average wrt all executions

• Assume we reserve 100 more elements each time
  - How many steps each insert takes on average?

• Intuitively: $\frac{n}{100}$. So **still** $O(n)$, same complexity!
  - Same for any **constant** number of empty elements $k$
  - Remember, complexity cares about large $n$! Think $n \gg k$
  - Can we do better?
Amortized-time complexity

• We see here the value of **amortized-time** complexity
  - A single execution can be slow
  - But “most” are fast
  - In many application we only care about the **average** wrt all **executions**

• Assume we reserve 100 more elements each time
  - How many steps each insert takes on average?

• Intuitively: $\frac{n}{100}$. So **still** $O(n)$, same complexity!
  - Same for any **constant** number of empty elements $k$
  - Remember, complexity cares about large $n$! Think $n \gg k$
  - Can we do better?
How to improve the complexity

• **Idea**: the number of empty elements must depend on $n$
  - Use more empty elements as the Vector grows!

• Standard approach: reserve $a \cdot n$ extra elements
  - for some constant $a > 1$, called the **growth factor**

• Common values
  - $a = 2$
  - $a = 1.5$

• In this class we will use $a = 2$
  - we always **double** the capacity
A property to remember

• Consider the **geometric progression** with ratio 2

\[ 1, 2^1, 2^2, \ldots, 2^n \]

• Summing \( n \) terms, we get the **next one minus 1**

\[ 1 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1 \]

• So each term is **larger** than all the previous together!

  - This is important since several quantities **double** in data structures
From linear to constant time

- We always **double** the capacity
  - What is the amortized-time complexity of insert?
- We do $n$ insertions starting from an empty Vector
  - Assume the last one was “slow” (the most “unlucky” case)
- How many **steps** did we perform **in total**?
  - $n$ steps just for placing each element
  - $n$ steps for the **last resize**
  - How many for all the previous resizes together?

\[
\frac{n}{2} + \frac{n}{4} + \ldots + 1 = n - 1
\]

- So less than $3n$ in total!
  - On average: $\frac{3n}{n} = O(1)$
- Key point: previous inserts are insignificant compared to the last one
Removing elements

- What about `vector_remove_last`?
- Simplest strategy: just consider the removed space as “empty”
  - `vector_remove_last` is clearly worst-case $O(1)$
  - Insert is not affected (we never reduce the amount of free space)
- Commonly used in practice
  - eg. `std::vector` in C++
- **Problem**: wasted space
Recovering wasted space

• **Idea**: if half of the array becomes empty, resize
  - the opposite of the doubling growing strategy
  - Is this ok?

• Careful
  - this is ok if we only remove
  - but a combination of remove+insert might become slow!

• Think of the following scenario
  - Insert \( n \) elements with \( n = 2^k \)
  - The vector is now full
  - Perform a series of: insert, remove, insert, remove, ...
Recovering wasted space

• **Idea**: if half of the array becomes empty, resize
  - the opposite of the doubling growing strategy
  - Is this ok?

• Careful
  - this is ok if we only remove
  - but a combination of remove+insert might become slow!

• Think of the following scenario
  - Insert \( n \) elements with \( n = 2^k \)
  - The vector is now full
  - Perform a series of: insert, remove, insert, remove, …
Recovering wasted space

• **Better strategy**
  - when only $\frac{1}{4}$ of the array is full
  - resize to $\frac{1}{2}$ of the capacity!
  - So we still have “room” to both insert and remove

• We can show that even a combination of insert+remove is $O(1)$ amortized-time
Implementation

Types

// Ένα VectorNode είναι pointer σε αυτό το struct.

```c
struct vector_node {
    Pointer value; // Η τιμή του κόμβου.
};
```

// Ένα Vector είναι pointer σε αυτό το struct

```c
struct vector {
    VectorNode array; // Τα δεδομένα, πίνακας από struct ve
    int size; // Πόσα στοιχεία έχουμε προσθέσει
    int capacity; // Πόσο χώρο έχουμε δεσμεύσει
    DestroyFunc destroy_value; // Συνάρτηση που καταστρέφει ένα στοι
};
```
Vector vector_create(int size, DestroyFunc destroy_value) {
  // Αρχικά το vector περιέχει size μη-αρχικοποιημένα στοιχεία, αλλά
  // δεσμεύουμε χώρο για τουλάχιστον VECTOR_MIN_CAPACITY για να απο
  // πολλαπλά resizes
  int capacity = size < VECTOR_MIN_CAPACITY ? VECTOR_MIN_CAPACITY :
  // Δέσμευση μνήμης, για το struct και το array.
  Vector vec = malloc(sizeof(*vec));
  VectorNode array = calloc(capacity, sizeof(*array));  // αρχικοπο

  vec->size = size;
  vec->capacity = capacity;
  vec->array = array;
  vec->destroy_value = destroy_value;

  return vec;
}
Random access is simple, since we have a real array.

```c
Pointer vector_get_at(Vector vec, int pos) {
    return vec->array[pos].value;
}

void vector_set_at(Vector vec, int pos, Pointer value) {
    if (value != vec->array[pos].value && vec->destroy_value != NULL) {
        vec->destroy_value(vec->array[pos].value);
    }
    vec->array[pos].value = value;
}
```
Insert, we just need to deal with resizes.

```c
void vector_insert_last(Vector vec, Pointer value) {
    // Μεγαλώνουμε τον πίνακα (αν χρειαστεί), ώστε να χωράει τουλάχιστον στοιχεία. Διπλασιάζουμε κάθε φορά το capacity (σημαντικό για την πολυπλοκότητα!)
    if (vec->capacity == vec->size) {
        vec->capacity *= 2;
        vec->array = realloc(vec->array, vec->capacity * sizeof(*new_)
    }

    // Μεγαλώνουμε τον πίνακα και προσθέτουμε το στοιχείο
    vec->array[vec->size].value = value;
    vec->size++;
}
```
Takeaways

- **Dynamic arrays** are the standard way to implement ADTVector

- Insert is $O(1)$
  - but **amortized-time**!
  - would you use a dynamic array in the software controlling an Airbus?

- Remove is also $O(1)$
  - also amortized, if we care about recovering wasted space

- Random access (get/set) is always worst-case $O(1)$